"SEISMIC ANALYSIS OF MULTI-STOREY SHEAR WALL BUILDINGS"

by

W.K. Tso (I) and J.K. Biswas (II)

SYNOPSIS

A seismic analysis is made on a typical multi-storey shear wall building. The natural frequencies and mode shapes of the building are determined by analysing an equivalent coupled shear wall system. Response spectrum technique is used to estimate the moment and shear in the structure. Comparisons are made with the code requirements. In addition, a dynamic analysis is performed to determine the moment and shear of the structure when it is subjected to a ground acceleration record.

GLOSSARY OF TERMS

A_1, A_2	- cross sectional areas of the shear walls
A_b	- cross sectional area of connecting elements
E	- Young's Modulus
F_{ij}	- equivalent static lateral load at ith floor due to mode j
G	- shear modulus
H	- height of building
I_1, I_2	- moments of inertia of shear walls
I_b	- moment of inertia of connecting element
$M_1(x), M_2(x)$	- moments at shear walls
Q_i	- shear force in the connecting element at ith floor
$[Sa]_j$	- acceleration response spectrum value for mode j
a	- distance between centerlines of shear walls
c	- length of connecting element
d_b	- depth of floor
g(t)	- ground motion
h	- storey height

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m_i - mass of the ith floor

$q(x,t)$ - distributed vertical shear in the connecting laminae

t - time

x - spatial coordinate

$y(x,t)$ - deflection of the shear walls

α_j - jth mode modal participation factor

$\phi_j(x)$ - jth mode shape

ϕ_{ij} - displacement of ith floor due to jth mode

ρ - average mass per unit height of building

ω_j - natural frequency

Ω - frequency ratio

INTRODUCTION

Many high rise apartment buildings are of the shear wall construction. Structurally this type of construction consists of parallel sets of shear walls coupled through the floor slabs of the building. The shear walls serve as load bearing walls for the vertical loads and also provide lateral resistance to wind or seismic loads.

Consider a multi-storey shear wall building with typical floor plan as shown in Fig. 1. If the asymmetry of the plan is small, the behaviour of the building subjected to lateral load in the weak direction may be found by analysing the behaviour of a typical pair of coupled shear walls of the building, as shown in Fig. 2. Based on observations of the performance of buildings of this type subjected to earthquakes [1], the damages occur most frequently near the base of the shear walls and at the connecting beams between the walls. Therefore, from a designer's point of view, the parameters of interest are the moments and shears at the base of the shear walls and the distribution of shear at the middle of the connecting beams along the height of the structure.

Static analysis of plane coupled shear walls subjected to lateral loading has been studied by many authors. One method of analysis used is the "continuum method". This method consists of replacing the discrete connecting elements between the shear walls by an equivalent system of laminae distributed throughout the height of the building. Solutions for the base moment at the walls and shear at the connecting beams have been obtained for the cases of uniformly distributed loads, triangular distributed loads and point loads [2,3,4]. Based on the same method, a dynamic study of coupled shear walls has been made to give the natural frequencies and mode shapes of the system [5].

In the present study, a seismic analysis is made on a typical multi-storey shear wall building. Firstly, the paper considers the effect of different geometric parameters of the building on its dynamic characteristics. Secondly, based on the dynamic characteristics of the building, the response spectrum technique [6] is used to estimate the designed moments and shears in the structure. A comparison is then made with code provisions to indicate the ductility requirement expected of such type of structure. Finally, the dynamic response of such a structure is computed based on the 1940 El Centro earthquake record. A comparison between the maximum dynamic response and the values calculated based on the spectrum technique is made. The importance of higher modal responses for this type of building is studied by comparing responses based on a single mode representation and responses based on the first three modes of structure.

STATEMENT OF PROBLEM

Consider a multi-storey shear wall building with a typical floor plan and dimensions as shown in Fig. 1. The building is subjected to ground motions g(t) along its weak direction. Assuming rigid in-plane diaphragm action of the floors, the response of the building can be obtained by considering a typical pair of shear walls coupled together by the floors. In this coupled shear wall representation of the building as shown in Fig. 2, the connecting elements are taken to have the same depth as the floors. The effective width of the floor is obtained based on the curves of Qadeer and Stafford Smith [7]. The mass of the building is proportionally divided among the sets of coupled shear walls.

Using the continuum method, the discrete connecting elements between the walls are replaced by a continuum distribution of independently acting laminae, each with depth dx, moment of inertia $(I_b dx)/h$, and cross-sectional area $(A_b dx)/h$. The idealized structure for the purpose of analysis is as shown in Fig. 3. Assuming the deflections of each wall to be the same so that the mid-points of the laminae are points of contraflexure, the equation of motion of the system can be simplified to the form [5].

$$E(I_1 + I_2)y^{IV} - \beta y'' + \rho\ddot{y} - \nu\rho\int_x^H \ddot{y}(\xi - x)d\xi = -\rho\ddot{g}[1 - \frac{\nu(H-x)^2}{2}] \tag{1}$$

where prime denotes differentiation with respect to the spatial variable x and dot denotes differentiation with respect to time t,

$$\nu = \frac{12J}{c^3}\left(\frac{A_1 + A_2}{A_1 A_2}\right) \tag{2a}$$

$$\beta = \nu E I_{eq} \tag{2b}$$

$$I_{eq} = \frac{A_1 A_2}{A_1 + A_2} \cdot a^2 + I_1 + I_2 \tag{2c}$$

and

$$J = \frac{I_b}{h[1 + 12EI_b/c^2 GA_b^*]} \tag{2d}$$

Once the deflection y(x,t) is determined, other parameters of interest can be found. The moment at the base of the wall M_j is given by

$$M_j = [EI_j y'']_{x=0} \qquad (j=1,2) \tag{3}$$

The distributed shear at the mid-points of the connecting laminae is given by

$$q(x,t) = [E(I_1 + I_2)y''' - \int_x^H \rho\ddot{y}(\xi,t)d\xi - \rho\ddot{g}(H - x)]/a \tag{4}$$

Finally, the shear at any discrete connecting element at a height x_j is given by

$$Q_j = \int_{x_j-h/2}^{x_j+h/2} q(x,t)dx \tag{5}$$

DYNAMIC CHARACTERISTICS CALCULATION

The natural frequencies and mode shapes can be found from the free vibration study. Taking $g(t) \equiv 0$ and seeking a solution of the form

$$y(x,t) = \phi_j(x)e^{i\omega_j t} \tag{6}$$

to equation (1), the jth mode shape $\phi_j(x)$ and natural frequency ω_j can be obtained from a solution of the eigenvalue problem

$$E(I_1 + I_2)\phi_j^{VI} - \beta\phi^{IV} - \rho\omega_j^2[\phi_j'' - \nu\phi_j] = 0 \tag{7}$$

The boundary conditions to be satisfied are

$$\phi_j(0) = 0 \tag{8a}$$

$$\phi_j'(0) = 0 \tag{8b}$$

$$\phi_j''(H) = 0 \tag{8c}$$

$$E(I_1 + I_2)\phi_j'''(0) - \rho\omega_j^2 \int_o^H \phi_j(\xi)d\xi = 0 \tag{8d}$$

$$E(I_1 + I_2)\phi_j^{IV}(H) - \beta\phi_j''(H) - \rho\omega_j^2\phi_j(H) = 0 \tag{8e}$$

$$E(I_1 + I_2)\phi_j^{V}(H) - \beta\phi_j'''(H) - \rho\omega_j^2\phi_j'(H) = 0 \tag{8f}$$

A detailed derivation of these boundary conditions and the technique of obtaining the natural frequencies and the corresponding mode shapes is given in reference [5]. The mode shape is of the form

$$\phi_j(x) = C_1\cosh\lambda_1 x + C_2\sinh\lambda_1 x + C_3\cosh\lambda_2 x + C_4\sinh\lambda_2 x$$
$$+ C_5\cos\lambda_3 x + C_6\sin\lambda_3 x \tag{9}$$

$C_1, C_2 \ldots C_6$ are constants determined up to an arbitrary constant; λ_1^2, λ_2^2 and λ_3^2 are the the roots of the characteristic polynominal

$$E(I_1 + I_2)\lambda^6 - \beta\lambda^4 - \rho\omega_j^2(\lambda^2 - \nu) = 0 \quad (10)$$

Based on the method indicated above, the dynamic characteristics of a 24 storey shear wall building are calculated. The floor plan and dimensions of the building are taken as shown in Fig. 1. The floor thickness is taken to be 6 inches with storey height of 9 feet. The shear walls have a thickness of 9 inches and the corridor width is taken to be 6 feet. The first five frequencies and associated mode shapes are determined as shown in Fig. 4.

By keeping the overall dimensions the floor plan constant, the effect of the variation in the width of the corridor, the floor thickness and the number of storeys on the natural frequencies are examined. The results are shown in Fig. 5 to Fig. 9.

In Fig. 5 is shown the variation of the first three natural frequencies as a function of the corridor width. The variations in frequencies are expressed in terms of a frequency ratio Ω which is defined as the building frequency normalized to the corresponding frequency with the building in its basic dimensions. Increase of corridor width results in a reduction of natural frequencies because both the rigidities of the shear walls and the connecting beams are reduced. In terms of percentage change, the fundamental frequency is most susceptible to corridor width variation.

Fig. 6 shows the first three natural frequencies as a function of floor thickness variations. Physically, an increase in floor thickness results in an increase of the mass of the building as well as an increase of the stiffness of the connecting elements. Therefore, the variation of the natural frequencies does not follow a fixed trend. The fundamental frequency increases while the third frequency decreases with increase in floor thickness. The second frequency is relatively insensitive to the change in thickness of the floor.

Fig. 7 shows that an increase of the number of storey decreases the natural frequencies. The fundamental frequency is least sensitive to the variation among the three frequencies computed. A comparison between the computed fundamental period and those calculated based on empirical formulae from the NBC 1970 code is given in Fig. 8. It can be seen that neither formulae from the code is accurate over the whole range of building heights considered.

One of the common assumptions used in analysing buildings with coupled shear walls is that the effect of axial deformation of the shear walls can be neglected. However, neglecting the axial deformation of the shear walls lead to an overestimation of the natural frequencies. In particular, the fundamental frequency may be overestimated by 10-20%, depending on the stiffness of the connecting beams. Fig. 9 shows the amount of overestimation of the frequencies as a function of the corridor width of the building.

RESPONSE ESTIMATE

Knowing the natural frequencies and the corresponding mode shapes, the seismic elastic response of the building can be estimated by means of the response spectrum technique. In the present study, the contribution of the first three modes to the seismic response is considered.

The response estimate consists of two steps. Firstly, based on the response spectrum technique, the static equivalent lateral forces corresponding to each mode are found. Assuming the masses of the building are concentrated at the floor levels, the equivalent lateral loads constitute a set of discrete forces acting at the floor levels of the building. The base moment, base shear of the shear walls and the shear force in the connecting elements are then determined using the continuum method of analysis.

Let ϕ_{ij} represent the displacement of the ith floor considering the jth mode of response. Therefore,

$$\phi_{ij} = \phi_j(x) \Big|_{x=ih} \qquad j = 1,2,3 \quad j = 1,2 \ldots\ldots 24 \tag{11}$$

The modal participation factor α_j for the jth mode can be found as [8]

$$\alpha_j = \sum_{i=1}^{24} m_i \phi_{ij} \Big/ \sum_{i=1}^{24} m_i \phi_{ij}^2 \tag{12}$$

The effective lateral seismic force generated on the ith mass due to the jth mode is then given by

$$F_{ij} = m_i \phi_{ij} \alpha_j \ [S_a]_j \tag{13}$$

where $[S_a]_j$ denotes the value of the acceleration response spectrum corresponding to the jth mode. For the purpose of calculation, the 1940 El Centro 2% damped response spectrum is used. The equivalent static load at the floor levels due to the first three modes are given in Table 1.

Once the equivalent static lateral loading is determined, the base moment, base shear of the shear walls and the shear forces in the connecting elements due to each mode can be found [9]. Plotted in Fig. 10 is the distribution of the shear q in the connecting elements along the height of the building due to the action of different modes. The shear force and bending moments at the base of the shear walls due to each mode are also shown in the same figure.

The root mean square values of the shear distribution q due to the first three modes are given in Fig. 11. Plotted on the same graph is the shear distribution considering the fundamental mode only. It can be seen that the one mode representation gives values of q similar to the three mode analysis. The higher mode contributions to q are concentrated towards the top of the building.

As a separate comparison, the shear distribution q is calculated based on the National Building Code of Canada, 1970. The base shear according to the code is given by

$$V = \frac{R}{4} \ . \ I.F.K.C.W. \tag{14}$$

The seismic factors involved in the base shear calculation are taken as follow: R = 4 (zone 3 loading); I = F = 1, K = 1.33, $C = 0.05(T)^{-1/3}$, $T = 0.05H/\sqrt{D}$, and W = 2720 kips. The base shear calculated according to equation (14) is 157 kips. The distribution of lateral loads at floor level is taken to be triangular plus a concentrated load on top of the building.

Fig. 11 shows that substantial difference exists between the shear distribution calculated according code and spectrum technique. The maximum distributed shear according to code is 1.9 kip/ft. while the corresponding value according to the spectrum technique is 6.6 kip/ft. The reduction factor in this case is 0.29, corresponding to a ductility factor of 3.5. It is interesting to observe that all curves predict the maximum around the mid-height of the building.

The actual response of the structure subjected to the actual 1940 El Centro ground record is also computed, using equation (1) and a three mode representation of the deflection variable y. The shear distribution q is then calculated using equation (4). A sample of the time history of q at the 4th floor and 16th floor level are shown in Fig. 12 and 13. It should be noted that the maximum does not occur at the same instant of time. Also, the time history record on the sixteenth floor shows less high frequency component than that on the fourth floor.

The maximum value of the shear distribution along the height of the building is shown in Fig. 11. A comparison between the q distribution as determined by spectrum analysis and that determined from complete dynamic analysis shows that the spectrum technique curve is conservative for design purposes in estimating the shear distribution in the connecting elements in the upper part of the building. In the lower part of the building, the shear as determined from the dynamic analysis is about 80% larger than that based on spectrum analysis.

A comparison is made on other quantities of interest, namely, maximum deflection at the top, the base shear and base moment on the shear walls. Shown in Table 2 are the results based on four methods of analysis. It can be seen that the values as determined by code are in general 3 to 4 times less than those obtained by the spectrum technique and the dynamic analysis. While the estimate based on the spectrum technique is conservative as far as deflection and base moment is concerned, the spectrum technique underestimates the base shear in the shear walls.

CONCLUSIONS

A seismic analysis is performed on a twenty-four storey shear wall building subjected to ground motion along its weak direction. The quanities of interest are the top deflection of the building, the base shear and base moment of the shear walls, and the shear distribution in the connecting elements between the shear walls along the height of the building. Three methods of calculation are used. The first method is based on the 1970 National Building Code of Canada recommendation. The second method is the use of the response spectrum curves to obtain the equivalent static lateral loading and the quantities of interest are calculated based on this equivalent static load. The third method is a complete dynamic analysis of the structure, treating the structure as a coupled shear wall system. The 1940 El Centro earthquake record and its response spectrum are used in the computation.

Based on the above study, the following conclusions are drawn:

1. The response spectrum technique is suitable to give seismic estimates on the base moment of the shear wall and the maximum deflection of the structure. Also, the spectrum method provides conservative estimate of the shear

distribution in the connecting elements in general. However, it underestimates the shear distribution in the lower portion of the building as shown in Fig. 11. Also, value of the base shear at the shear walls calculated using the spectrum method is lower than that determined by dynamic analysis. Therefore, appropriate compensation measure should be adopted if the spectrum technique is used to determine the base shear of shear wall buildings.

2. Inclusion of higher mode contribution in the spectrum method amounts to a 20-30% increase in the value of base shear and base moment in the shear walls. The contribution to the connecting beam shear is small except towards the top of the building.

3. The calculations based on code recommendation are in general one-third to one-fourth of those based on the spectrum method or the dynamic analysis. If the equal deflection concept of equivalence between elastic and inelastic responses is used, this implies a ductility factor of 3 to 4 for the resisting elements in the shear wall building. Appropriate measures in design and construction to provide this amount of ductility become necessary.

4. A study on the natural frequencies of the building shows that the formula for period determination as suggested by the code may lead to errors of 20% or more. A separate calculation for the natural frequencies is necessary for this type of building. The mode shapes as determined in the free vibrational analysis are very close to those of a uniform cantilever. This fact may be used to facilitate the determination of the natural frequencies by means of the Rayleigh-Ritz approximate technique.

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TABLE - 1

MODAL FORCES FROM SPECTRUM METHOD

FLOOR	MODE - 1	MODE - 2	MODE - 3
24	47.2 (kips)	-71.7 (kips)	48.8 (kips)
23	44.9	-58.3	33.3
22	42.5	-44.9	18.0
21	40.2	-31.4	3.4
20	37.8	-18.4	- 9.6
19	35.4	- 5.7	-20.3
18	32.9	6.4	-27.8
17	30.5	17.4	-31.7
16	28.0	27.3	-31.6
15	25.6	35.6	-27.8
14	23.1	42.2	-20.8
13	20.7	47.0	-11.3
12	18.3	49.9	- 0.4
11	15.9	50.8	10.7
10	13.6	49.8	20.9
9	11.5	47.0	29.0
8	9.4	42.7	34.3
7	7.4	37.2	36.4
6	5.6	30.7	34.9
5	4.0	23.7	30.6
4	2.7	16.8	23.8
3	1.6	10.4	15.9
2	0.7	5.0	8.2
1	0.2	1.4	2.4

TABLE - 2

COMPARISON OF DEFLECTION, BASE SHEAR
& SHEAR WALL MOMENT

	DEFLECTION AT TOP (inch)	BASE SHEAR FORCE ON ONE WALL (kip)	BASE MOMENT ON ONE WALL (kip ft.)
Equivalent Static Loading according to Code	2.9	78.5	8160
Static Loading according to Response Spectrum (Single Mode)	9.4	249.8	25110
Static Loading according to Response Spectrum (RMS of 3 Modes)	9.7	306.3	33650
Dynamic Analysis using 3 Modes	6.9	350.0	20000

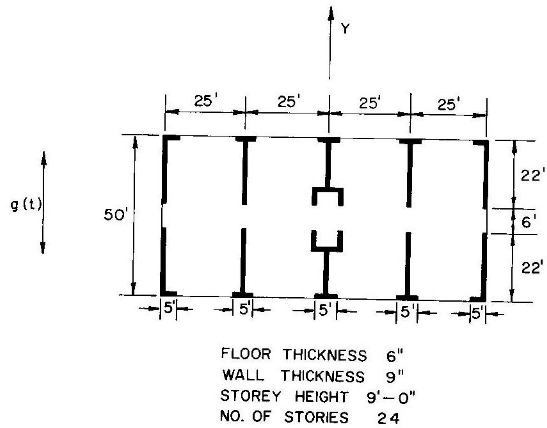


FIGURE. 1 FLOOR PLAN

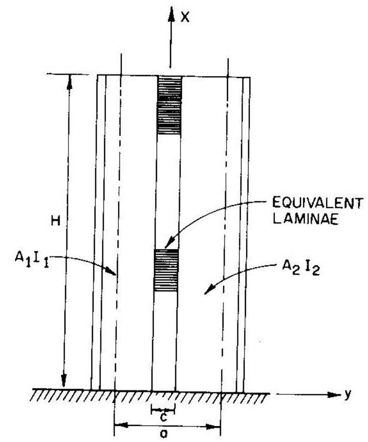


FIGURE. 3 EQUIVALENT MODEL FOR CONTINUUM METHOD

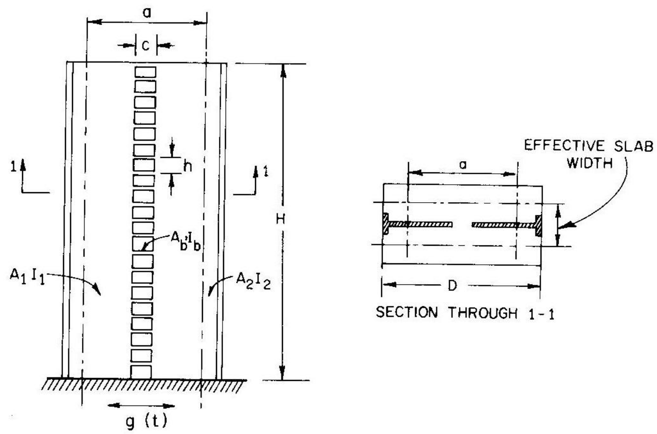


FIGURE. 2 COUPLED SHEAR WALL UNIT

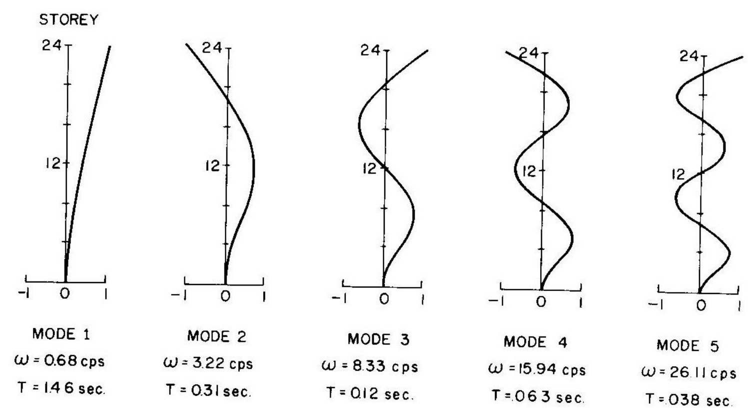


FIGURE. 4 MODAL CHARACTERISTIC OF BUILDING IN BASIC CONFIGURATION

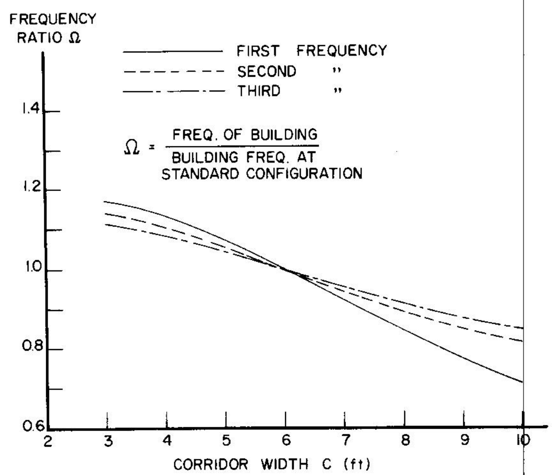


FIGURE. 5 EFFECT OF CORRIDOR WIDTH

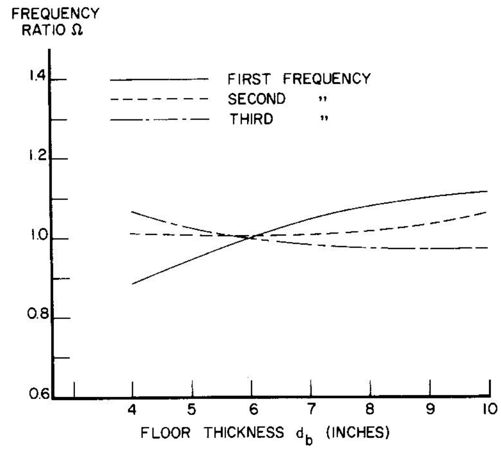


FIGURE. 6 EFFECT OF FLOOR THICKNESS

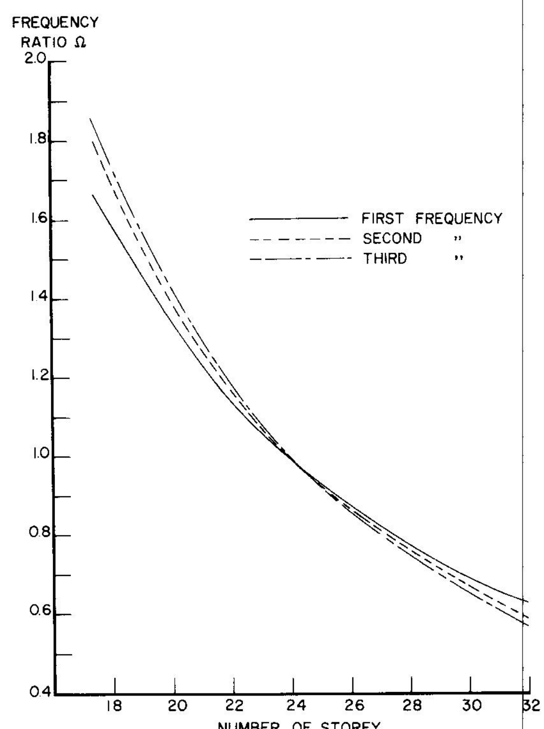


FIGURE. 7 EFFECT OF NUMBER OF STOREY

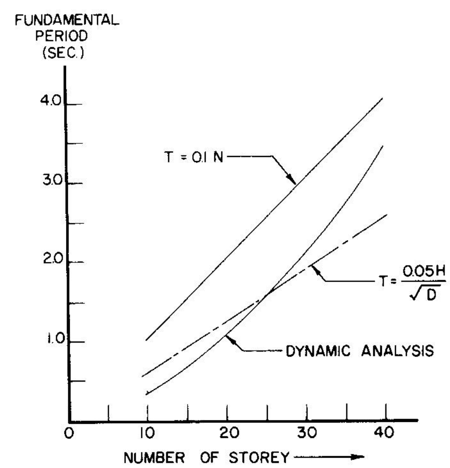


FIGURE. 8 COMPARISON OF FUNDAMENTAL PERIOD

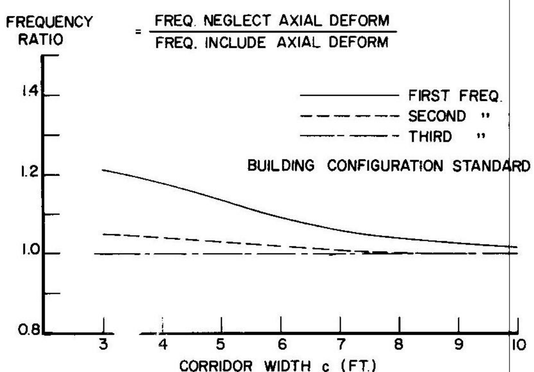


FIGURE. 9 EFFECT OF NEGLECTING AXIAL DEFORMATION IN WALLS

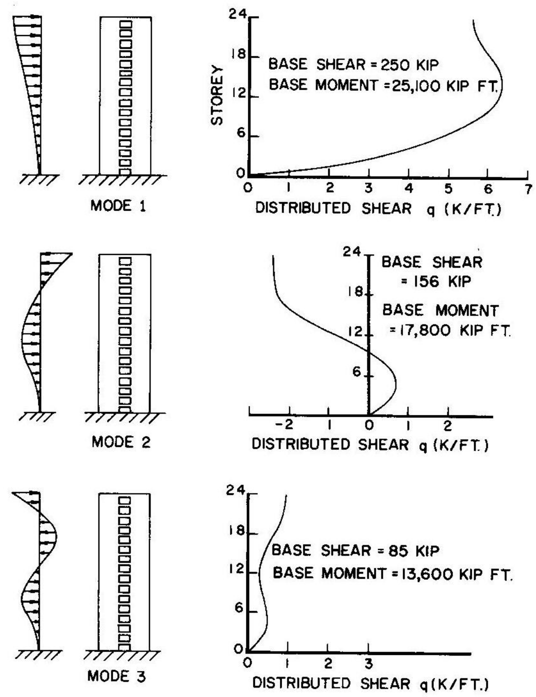


FIGURE. 10 SHEAR DISTRIBUTION IN DIFFERENT MODES

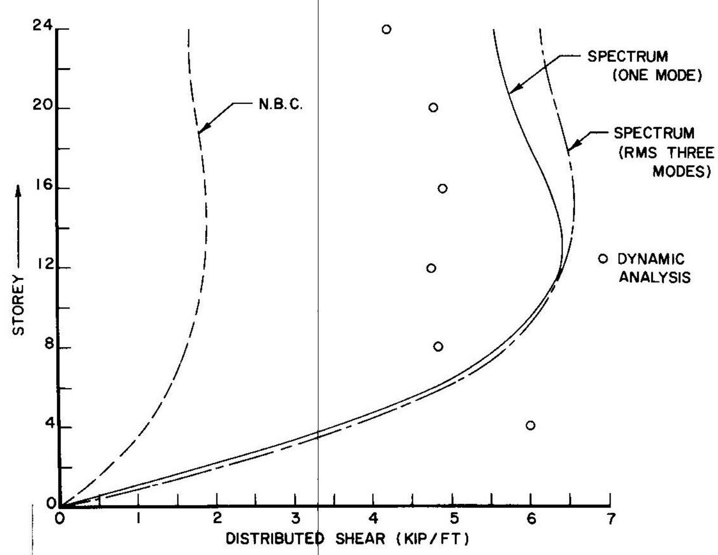


FIGURE. 11 COMPARISON OF DISTRIBUTED SHEAR IN CONNECTING BEAM

FIGURE. 12 DISTRIBUTED SHEAR AT 4 th FLOOR LEVEL

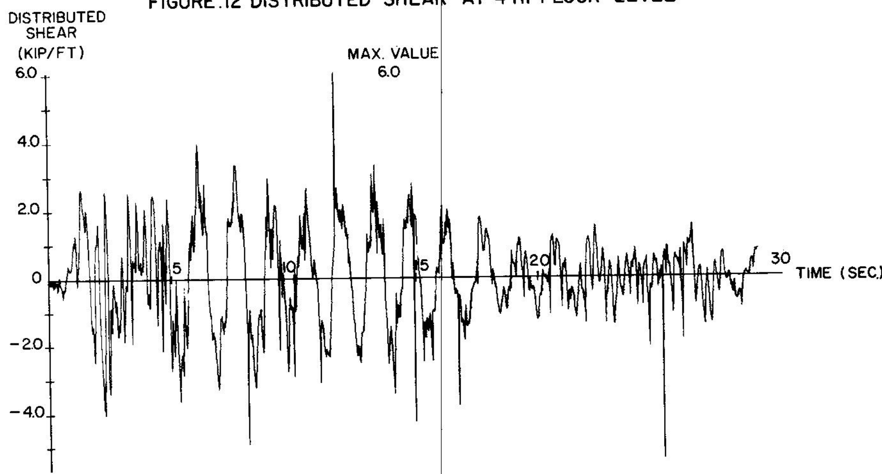
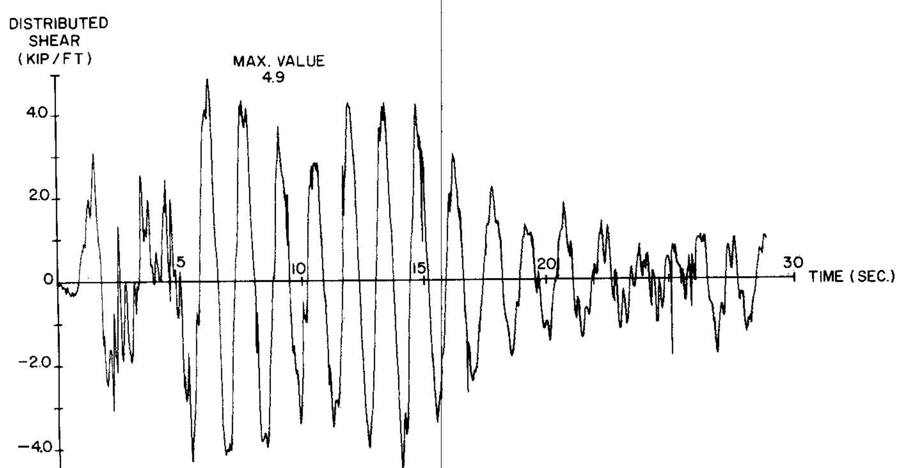


FIGURE. 13 DISTRIBUTED SHEAR AT 16 th FLOOR LEVEL



DISCUSSION OF PAPER NO. 14

DYNAMIC RESPONSE OF MULTI-STOREY SHEAR WALL BUILDINGS

by

W.K. Tso and J.K. Biswas

Discussion by: J.H. Rainer

The magnitudes of computed forces from the dynamic analysis is fairly sensitive to the level of damping assumed. The damping of 2% of critical assumed appears to be on the low side. By taking a damping of 3 or 4% of critical the difference between the computed values and those obtained from the N.B.C. would be substantially reduced.

Reply by: W.K. Tso

The author agrees with the observation made by Dr. Rainer. By using a higher value of damping, such as 5% critical, the computed forces from the dynamic analysis will be reduced to about 3/4 of the values computed using 2% critical damping. This will make the computed values about 2 to 3 times those obtained from the N.B.C.

Question by: R.A. Spencer

What does a "ductility factor of 3 or 4" mean in the context of a shear wall building like that described here?

Reply by: W.K. Tso

"A ductility factor of 3 or 4" as used in the paper means that elements in the shear wall building designed according to the National Building Code of Canada should be capable of sustaining deformation three or four times that of yield deformation without loss of load carrying capability. This is the same concept of ductility requirement as commonly applied to frame structures. The purpose of the paper is to apply this concept to shear wall buildings and establish the ductility requirements for this type of structure.

Question by: J.G. MacGregor

Paulay's analysis* of the ductility required in connecting beams suggests that these members will need a ductility factor of 12 - 20 if the entire shear wall is to have a ductility of 3 - 4. Tests by Paulay**, Bertero*** and others show that such elements do not have this ductility under repeated cycles of reversed loads. It should be noted, however, that the slab elements considered by the authors would tend to be much more ductile than the beams considered by Paulay provided that problems of shear lag in the slabs did not result in cracks in the slabs near the walls, thus reducing their effectiveness.

* ACI Journal, Nov. 1970
** ASCE Proc. Str. Div., Fall 1970
*** Reported at Denver ACI Mtg., 1971.

Reply by: W.K. Tso

The author appreciates the comment made by Professor MacGregor. Paulay's study is concerned with the deep connecting beams at the end walls of the building while the depth of the connecting beams in the present analysis is taken to be the thickness of the floor slab of the building. This may be the cause of the difference when Paulay suggests a ductility factor of 12 - 20 for his study of deep connecting beams while the present analysis suggests a value of 3 - 4. In a multi-storey building, there are intermediate sets of shear walls connected through floor slabs and end shear walls connected by deep connecting beams. The ductility requirements on the connecting elements between the walls would be different depending on whether one is concerned with the floor slabs or the deep connecting beams.

The purpose of the paper is to present a method to establish the ductility requirements for the shear wall type structure. The question of achieving the ductility requirements as raised by Professor MacGregor is an important question not examined in the paper.